Day. 1904.	R.A. Time.	R.A. Arc.	Dec.	Day. 1904.	R.A. Time.	R.A. Arc.	Dec.
	s	"	"		s	"	"
Oct. 15	−·05	−0·7	+0·3	Nov. 24	−·06	−0·8	−0·1
23	−·06	−0·9	+0·1	Dec. 2	−·05	−0·7	−0·1
31	−·06	−0·9	0·0	10	−·06	−0·8	−0·1
Nov. 8	−·06	−0·9	0·0	18	−·07	−1·0	−0·2
16	−·07	−1·0	+0·1	26	−·05	−0·7	−0·3

Experimental Reduction of some Photographs of Eros made at the Cambridge Observatory for the Determination of the Solar Parallax. By Arthur R. Hinks, M.A.

1. The long series of photographs of the planet *Eros* which I obtained last winter with the new Sheepshanks equatorial of the Cambridge Observatory was very much interrupted by bad weather. My programme was to make four exposures on the planet every hour between 6^h E. and W.—the construction of the instrument precludes greater hour-angles—and, of each set of four, two were made following the stars and letting the planet trail, two following the planet and letting the stars trail. There was only one case of two consecutive nights clear, or nearly so, right through, viz. 1900 November 9 and 10, and the plates taken on these nights, when the planet was but little past opposition, and the parallax factor had become fairly large, were clearly indicated as the subject of preliminary and experimental reductions. The plates were to be centered for each night on the position of the planet at Berlin midnight, the epoch of Professor Millosevich's ephemeris; but, by an accident very fortunate for our present purpose, the plates on the 9th were set to a centering very near that proper to the 10th, and the mistake was discovered in time to use the same point for the latter plates, so that all the exposures to be here treated have approximately the same centering on the sky.

2. As many as twenty exposures were made on one plate, the displacement necessary between each being given to the plate-carrier on rectangular slides on the breach-piece of the telescope, not by displacement of the pointing of the telescope on the sky. There is an obvious advantage in this, that the centre of projection remains unchanged, and all the exposures can be made comparable by linear formulæ of reduction, which is not strictly the case when the pointing of the telescope is changed.

3. The stars on the plates whose meridian places are known are not very symmetrically distributed about the locus of the planet, and many of them are inconveniently bright. It was necessary to consider the possibility of a systematic error due to the trail of either stars or planet—the motion of the planet was

more than a second of arc per minute—and if this exists it is almost necessarily a function of the magnitude. I selected, therefore, fourteen comparison stars well distributed about the locus of the planet, and of magnitudes as nearly as possible equal to it. These fourteen stars were measured with the planet on each exposure, with the new measuring machine which I have recently described (*Monthly Notices*, 1901 May). I may take this opportunity of saying that in accuracy, rapidity, and pleasure of use the machine has more than come up to expectation.

4. *Search for a systematic error due to trail.*—Each set of four exposures was arranged thus:—

a . . 3^m, b . . 2^m, stars fixed ; c . . 3^m, d . . 2^m, planet fixed.

A preliminary comparison showed that for any exposure the differences $b-a$ and $d-c$ for the fourteen comparison stars varied only accidentally from a constant; they could not be represented by a linear function of the coordinates of the stars; that is to say, during the short intervals the variations in differential refraction, orientation, &c., were insensible. In all that follows I have used the means of pairs, $\frac{1}{2}(a+b)$ and $\frac{1}{2}(c+d)$, as single exposures.

Next, an examination of the differences $\frac{1}{2}\overline{c+d}-\frac{1}{2}\overline{a+b}$ gave a similar result; they could not be represented by a linear expression. But they ought to show an effect of error due to trail varying with the magnitude—if this error exists, indeed, at all. To examine this point, four fainter stars, whose images were barely measurable, were added to the above list of fourteen and measured on all the plates. For each of thirteen sets of four exposures the differences $\frac{1}{2}\overline{c+d}-\frac{1}{2}\overline{a+b}$ were formed, their mean taken, and the values of mean−individual found. Then, taking the mean of the thirteen results for each star, we have:

	B.D.	Mean Residual.		B.D.	Mean Residual.
1	+54° 434	−2	10	+54° 449	+1
2	54 435	−3	11	53 444	+1
3	54 438	−1	12	54 451	−1
4	54 439	−3	13†	54 454	−1
5	54 442	−1	14†	Anon.	+11
6†	54 443	−2	15	Anon.	−3
7	Anon.	0	16	54 459	−2
8	54 445	−3	17†	53 449	+4
9	53 441	−1	18	54 461	−1

It does not seem worth while to add the visual magnitudes. The four stars, photographically fainter, which were added to the original fourteen are distinguished by a dagger. The

quantities are in ten-thousandths of a réseau interval, so that $1 = 0''{\cdot}017$. There is evidently something peculiar about star No. 14; it may be a close double. Its large positive residuals have made nearly all the others negative. For the three other faint stars the mean residuals are -2, -1, $+4$. It seems to me that there is no evidence of error depending upon trail varying with the magnitude; that it would certainly do so if it existed at all; and that it is unlikely therefore that it exists as a constant error. I propose, however, to keep the two series—A, stars fixed, B, planet fixed—separate throughout the work. The four faint stars, 6, 13, 14, 17, have not been further employed.

5. *Probable error of a measure.*—Incidentally we may derive from this discussion, if we assume that the differences $b-a$, $d-c$, and $\frac{1}{2}\overline{d+c}-\frac{1}{2}\overline{b+a}$ are entirely accidental and free from the systematic error depending on trail, an estimate of the P.E. of a measure on a single-star disc, *including the real error of position of the apparent centre of the photographic image itself.* Each measure includes two settings on the star disc in each of two positions of the plate differing in orientation by 180°. The results are :—

	From $b-a$.	$d-c$.	$\frac{1}{2}\overline{d+c}-\frac{1}{2}\overline{b+a}$.
P.E. of a complete measure in ξ	$\pm^{R}{\cdot}00053$	$\pm^{R}{\cdot}00065$	$\pm^{R}{\cdot}00060$
P.E. of a complete measure in η	$\pm\ {\cdot}00048$	$\pm\ {\cdot}00052$	$\pm\ {\cdot}00043$

$0{\cdot}^{R}0001 = 0''{\cdot}0175$. We may take as the P.E. of a measure of one image in ξ $\pm 0''{\cdot}10$, in η $\pm 0''{\cdot}08$.

6. *Reduction of all the plates to standard.*—The fourteen symmetrically arranged comparison stars were selected to provide a strong means of comparing the plates *inter se.* Besides the symmetry of their arrangement, they had the advantage of near equality in magnitude to the planet. Two "zero" exposures were chosen, one for each series, A and B, taken near the meridian on November 10, and all the others were first made comparable with these zero exposures. Following Turner's method of comparison by linear formulæ, equations of condition were formed of the usual pattern :

$$a\xi + b\eta + c + \xi - \xi' = 0$$
$$d\xi + e\eta + f + \eta - \eta' = 0$$

and for each exposure the six constants were found to reduce it to compare with a zero exposure.

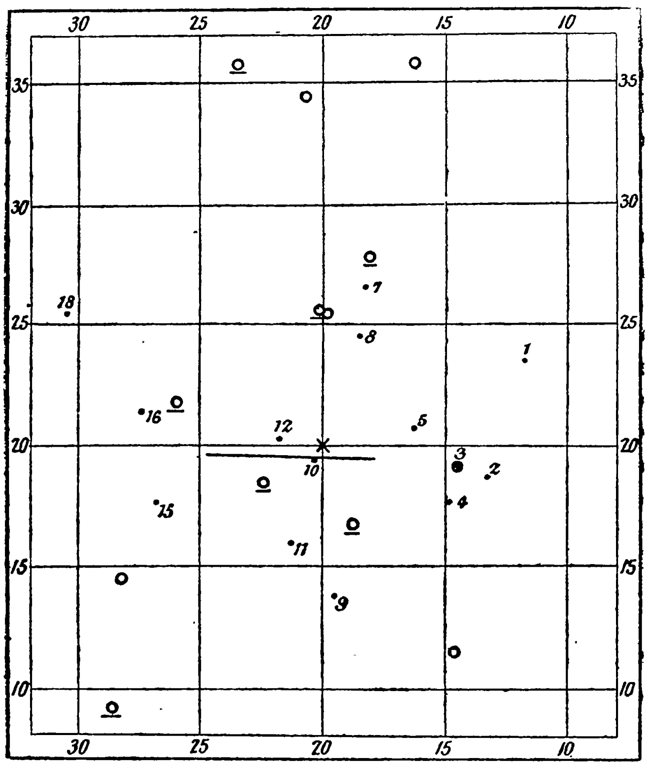
I have preferred to put aside for the present the well-disputed question whether the unsymmetrical part of the differential refraction should be calculated and applied numerically, and the six constants thereby reduced to four. The older method is shorter, and has some advantages in this first experimental work.

We have now each series of exposures comparable among

themselves, but affected still with the differential refraction and aberration, errors of orientation, scale-value, and centering of the zero exposures. On these two exposures only I have measured the images of all the stars on the plate which were to be found in the Harvard A. G. Zone Catalogue—there were scarcely enough stars in the Paris list, nor were they well distributed (see figure). The rectangular coordinates of these standard stars were computed for 1900·0 with assumed centre of plate

$$\alpha \quad 1^{h}\ 57^{m}\ 8^{s} \qquad \delta + 54^{\circ}\ 22'{\cdot}0$$

and the six constants to reduce each zero exposure to standard were found as above.



Centre of plate (1900·0) α 1h 57m 8s $\delta + 54^{\circ}$ 22′·0.
• Comparison star. ——— Path of planet.
o Standard star; underlined o if in Paris list.
Half scale. Every fifth R. line only shown. 5R. = 14′·6.

The reduction of all the exposures to standard for 1900·0 proceeded, therefore, in two steps:

(1) Each exposure was reduced to the zero of its series by the aid of constants derived from the fourteen comparison stars.

(2) All were then reduced from zero to standard by the constants derived from the thirteen meridian standard stars (of which only two had served also as comparison stars).

The advantages of this double procedure seem to me to be as

follows:—In addition to the fact that the comparison stars are well distributed, and nearly equal to the planet in magnitude, we have the further advantage that there is no question of meridian places in the first step. The exposures are made comparable with one another by a process that may be considered final. No revision of meridian places can affect it. On the other hand, the reduction from zero to standard depends on the accuracy of the coordinates of the standard stars calculated from their meridian places; it may well have to be revised when better places are available. When this is done there will be only two sets of equations to solve again, instead of twenty-six.

I do not, however, anticipate that any revision of meridian places will seriously alter my constants for the reduction to standard, except perhaps the errors of centering c and f, which will be affected by the systematic difference between the fundamental systems of Auwers and Newcomb. The average residuals in the reduction of the thirteen standard stars are in ξ 0″·52 and in η 0″·44, which speaks well for the original accuracy of the Harvard places, affected as they are now by unknown proper motions for about twenty-five years.

7. *Search for evidence of the effect of atmospheric dispersion.*—The possibility of systematic error due to atmospheric dispersion is an objection that will be alleged against the determination of the solar parallax by photography. To this may be added in the present case the possibility of variable distortion due to flexure of the plane mirror of the Cambridge equatorial varying with the hour-angle. The two effects may here be included in the category "hour-angle error" as used by Kapteyn. It is possible to find in the reduction of all the exposures to a zero near the meridian a little evidence on this matter. In series A I have taken six exposures, three on each side of the meridian, and formed for each star the differences of the reduced values of ξ from the values on the zero exposure. They are given in the following table, the unit as before being the ten-thousandth of a réseau interval.

	East.			West.			
Hour-Angle.	$-4^h·9$.	$-4^h·0$.	$-2^h·9$.	$+2^h·9$.	$+4^h·0$.	$+4^h·9$.	$\frac{1}{3}(\Sigma W-\Sigma E)$.
Star 1	0	− 5	− 3	+ 2	0	+ 3	+ 4
2	+ 5	− 2	− 9	− 5	+ 3	+ 2	+ 3
3	− 4	+ 6	+ 2	+ 10	+ 4	+ 1	+ 4
4	− 1	+ 4	+ 7	+ 2	− 4	+ 3	− 3
5	+ 2	+ 1	+ 4	+ 5	+ 2	+ 6	+ 2
7	− 12	− 11	− 13	− 11	− 12	− 14	0
8	+ 2	+ 1	0	− 2	− 3	− 2	− 3
9	+ 8	+ 6	+ 11	0	+ 9	0	− 5
10	− 6	− 11	− 11	+ 5	− 4	+ 2	+ 10
11	+ 4	0	+ 2	+ 1	+ 2	+ 12	+ 3

Hour-Angle.	East. $-4^{h}\cdot9$.	$-4^{h}\cdot0$.	$-2^{h}\cdot9$.	West. $+2^{h}\cdot9$.	$+4^{h}\ 0$.	$+4^{h}\cdot9$.	$\frac{1}{3}(\Sigma W - \Sigma E)$.
Star 12	+ 1	− 7	+ 4	− 1	+ 1	− 7	− 2
15	− 5	+ 4	− 7	−11	− 11	−13	− 9
16	0	− 4	+ 7	+ 7	+10	+ 1	+ 5
18	+ 8	+17	+ 5	− 1	+ 2	+ 5	− 8

If a star is extra blue its ξ is increased W. and diminished E.; the above residuals are algebraically diminished W. and increased E., so that the value of W.—E. (which is independent of the errors of measurement on the zero plate, except in so far as they affect the constants of reduction) is negative for an extra blue star. Taking the P.E. of a single measure in ξ as 6, the probable value of a quantity in the last column, allowing a little for accumulated errors in the reduction to zero, will be about 4—it must be remembered that each residual represents the mean of two exposures. Of the fourteen quantities in the last column, five are above 4, two equal to it, and seven below 4. There is, therefore, no indication that these quantities are other than purely accidental. The evidence is slight, but it is at least in the right direction.

8. *Reduced measures of the planet.*—The following table contains the mean epoch of exposure, expressed in decimals of a day from Berlin mean noon—called afterwards T+ΔT—the measured coordinates of the planet (means of *a* and *b* or of *c* and *d*), the same reduced to zero, and then to standard, and finally these last referred to the centre of the plate (20·0 20·0) as origin, and multiplied by the constant ·00085221 to express them in terms of the radius of projection.

Table I.

Measures of the planet.

	Series A.	Measures. ξ and η	Red. to Zero.	Red. to Standard.	In parts of Radius.
	d.				
1	Nov. 9·552382	24·8337	24·6146	24·5935	+·00391463
		19·9144	19·7026	19 6764	−·00027578
2	·605387	24·4737	24·2663	24·2453	+·00361789
		19·9812	19·6846	19·6564	−·00029282
3	·649279	24·2036	23·9821	23·9612	+·00337577
		20·1298	19·6663	19·6365	−·00030978
4	·688555	23·9549	23·7352	23·7144	+·00316545
		20·1703	19 6461	19·6149	−·0003281
5	·729413	23·7058	23·4828	23·4621	+·00295044
		20·2200	19·6207	19·5880	−·00035111
6	10·277103	20·2625	20·2907	20·2704	+·00023044
		19·8577	19·5107	19·4596	−·00046053

	Series A.	Measures. ξ and η	Red. to Zero.	Red. to Standard.	In parts of Radius.
	d.				
7	Nov. 10·317877	20·0017	20·0315	20·0112	+·00000954
		19·7320	19·5137	19·4612	−·00045917
8	·356861	19·7636	19·7814	19·7611	−·00020359
		19·6065	19·5121	19·4581	−·00046181
9	·464122	19·0770	zero	19·0568	−·00080380
		19·4933	exposure	19·4353	−·00048124
10	·515599	18·9717	18·7358	18·7157	−·00109449
		19·4730	19·4724	19·4124	−·00050076
11	·565637	18·6214	18·4073	18·3873	−·00137436
		19·5340	19·4453	19·3834	−·00052547
12	·605862	18·3782	18·1460	18·1261	−·00159696
		19·6375	19·4182	19·3548	−·00054976
13	·650159	18·0889	17·8614	17·8417	−·00183932
		19·7751	19·3849	19·3199	−·00057959
	Series B.				
1	Nov. 9·556968	24·6744	24·4636	24·5632	+·00388880
		19·9127	19·6972	19·6746	−·00027730
2	·609456	24·3153	24·1187	24·2184	+·00359496
		19·9880	19·6793	19·6547	−·00029427
3	·653376	24·0454	23·8368	23·9366	+·00335481
		20·1384	19·6605	19·6343	−·00031165
4	·693284	23·7892	23·5845	23 6844	+·00313988
		20·1689	19·6388	19·6112	−·00033134
5	·733597	23·5449	23·3372	23·4372	+·00292922
		20·2229	19·6170	19·5879	−·00035120
6	10·281374	20·1128	20·1415	20·2420	+·00020623
		19·8306	19·5071	19·4598	−·00046036
7	·322405	19·8471	19·8811	19·9815	−·00001577
		19·7276	19·5089	19·4601	−·00046011
8	·361507	19·6090	19·6310	19·7314	−·00022890
		19·6030	19·5080	19·4577	−·00046215
9	·468533	18·9275	zero	19·0280	−·00082835
		19·4876	exposure	19·4333	−·00048295
10	·519523	18·8336	18·5904	18·6910	−·00111554
		19·4762	19·4669	19·4107	−·00050221
11	·569935	18·4646	18·2580	18·3588	−·00139865
		19·5361	19·4384	19·3803	−·00052811
12	·610161	18·2281	17·9975	18·0984	−·00162056
		19·6397	19·4104	19·3508	−·00055325
13	·654831	17·9314	17·7113	17·8124	−·00186429
		19·7830	19·3769	19·3157	−·00058317

9. *Comparison with the ephemeris. General considerations.*—We are now in a position to compare with the ephemeris, and here we come to the essential point of what I have called this experimental reduction. It was suggested by Mr. Cowell in *The Observatory* (1900 December) that there are clear advantages in using in astrographic methods a heliocentric ephemeris of the planet expressed in rectangular coordinates. He points out that the reduction of the measured positions of stars and planet to standard, by the linear transformations, corrects both stars and planet completely for the aberration of light (excepting a small term depending on the eccentricity of the Earth's orbit). For the planet this is over-correction, inasmuch as the usual method of allowing for the motion of the planet during the time the light is on its way from it to us, is to leave it uncorrected for the aberration of light and to antedate the observation by the light-time. We thus obtain the true position of the planet at the antedated time, freed from both planetary aberration and the aberration of light. If, however, we correct automatically the planet with the stars for the aberration of light, we obtain the position of the planet at the time the light left it as seen from the Earth at the time the light reached it; and this is not immediately comparable with the ephemeris in its ordinary form. This difficulty is evaded if we proceed in the following manner, for the idea of which I must acknowledge my indebtedness to Mr. Cowell's paper.

10. Let the unit of distance be the mean distance of the Earth from the Sun, and let U, V, W be the heliocentric coordinates of the planet at the epoch T (the time the light left it) referred to three rectangular axes, Su, Sv, Sw. S is the centre of the Sun; the positive directions of the axes are Su towards the vernal equinox for 1900·0; Sv in the plane of the mean ecliptic for this epoch, at right angles to Su, towards Cancer; Sw towards the north pole of the ecliptic.

Now let f, g, h be the heliocentric coordinates referred to the same axes of the centre of the Earth at epoch $\mathrm{T}+\Delta\mathrm{T}$ (the time when the light which left the planet at T reaches the Earth). Transform to parallel axes through the centre of the Earth, E.

$\mathrm{U}-f$, $\mathrm{V}-g$, $\mathrm{W}-h$, are the coordinates of the planet at epoch T as seen from the centre of the Earth E at epoch $\mathrm{T}+\Delta\mathrm{T}$. (System I.)

Now transform to a new set of axes through the same origin Ex, y, z; Ez is drawn through the point on the celestial sphere corresponding to the assumed centre of the plate for 1900·0; Ey is at right angles to this in the plane of the meridian of the plate; Ex is at right angles to the others. Ex, Ey are then parallel to the axes of coordinates on the standard plate. (System II.)

If α, δ is the assumed centre of the plate, the direction cosines of the axes of System II. referred to the axes of System I. are easily found to be

$$
\begin{array}{lll}
l_1 = -\sin\alpha & m_1 = -\sin\delta\cos\alpha & n_1 = \cos\delta\cos\alpha \\
l_2 = \cos\epsilon\cos\alpha & m_2 = \sin\epsilon\cos\delta & n_2 = \sin\epsilon\sin\delta \\
 & \quad -\cos\epsilon\sin\delta\sin\alpha & \quad +\cos\epsilon\cos\delta\sin\alpha \\
l_3 = -\sin\epsilon\cos\alpha & m_3 = \cos\epsilon\cos\delta & n_3 = \cos\epsilon\sin\delta \\
 & \quad +\sin\epsilon\sin\delta\sin\alpha & \quad -\sin\epsilon\cos\delta\sin\alpha.
\end{array}
$$

And with respect to the axes of System II. the coordinates of the planet at epoch T, as seen from E at epoch $T+\Delta T$, are

$$
\begin{aligned}
x &= l_1\,(U-f)+l_2\,(V-g)+l_3\,(W-h) = L \\
y &= m_1\,(U-f)+m_2(V-g)+m_3(W-h) = M \\
z &= n_1\,(U-f)+n_2\,(V-g)+n_3\,(W-h) = N
\end{aligned}
$$

Again, let x_0, y_0, z_0 be the coordinates of the Observatory O at epoch $T+\Delta T$ referred to the axes of System II. It is easily seen that in terms of the mean distance R of Earth from Sun

$$
\begin{aligned}
x_0 &= \frac{\rho}{R}\cdot\frac{\rho'}{\rho}\cos\phi'\sin h &&= \pi a \\
y_0 &= \frac{\rho}{R}\cdot\frac{\rho'}{\rho}(\sin\phi'\cos\delta - \cos\phi'\sin\delta\cos h) &&= \pi b \\
z_0 &= \frac{\rho}{R}\cdot\frac{\rho'}{\rho}(\sin\delta\sin\phi' + \cos\delta\cos\phi'\cos h) &&= \pi c
\end{aligned}
$$

where h is the hour-angle of the centre of the plate, ϕ' the reduced latitude of the Observatory, ρ' and ρ the local and equatorial radii of the Earth, π the Sun's equatorial horizontal parallax.

Transform finally to another set of axes through O, parallel to the axes of System II.

Then
$$
\begin{aligned}
X &= L - a\pi \\
Y &= M - b\pi \\
Z &= N - c\pi
\end{aligned}
$$

are the coordinates of the planet at epoch T as seen from the Observatory at epoch $T+\Delta T$.

And the position of the planet projected upon the ideal standard plate will be

$$
\begin{aligned}
\xi_0 &= \frac{X}{Z} = \frac{1}{N}\left\{L - \left(a - \frac{c}{N}\right)\pi\right\} = \nu\{L-(a-c\nu)\pi\} \\
\eta_0 &= \frac{Y}{Z} = \frac{1}{N}\left\{M - \left(b - \frac{c}{N}\right)\pi\right\} = \nu\{M-(b-c\nu)\pi\}
\end{aligned}
$$

since πc is small compared with N; ν is written for $1/N$ to avoid fractions in the printing.

Now in forming the equations of condition we must put $\pi = \pi_0 + \Delta\pi$, and it will be necessary to introduce tabular corrections to the ephemeris position of the planet—it will be assumed for the present that the heliocentric ephemeris of the Earth is sufficiently good. If these corrections are ΔU, ΔV, ΔW, we have for the true ξ coordinate of the planet

$$\frac{L + l_1 \Delta U + l_2 \Delta V + l_3 \Delta W - a(\pi_0 + \Delta\pi)}{N + n_1 \Delta U + n_2 \Delta V + n_3 \Delta W - c(\pi_0 + \Delta\pi)}$$

$$= \nu\{L - \pi_0(a - \nu c)\} + \nu(l_1 \Delta U + l_2 \Delta V + l_3 \Delta W) - \nu(a - c\nu)\Delta\pi$$

which we will write

$$\xi = \xi_0 + \Delta\xi_0 - \nu(a - c\nu)\Delta\pi$$

and similarly

$$\eta = \eta_0 + \Delta\eta_0 - \nu(b - c\nu)\Delta\pi$$

Hence each reduced measure (ξ, η) of the planet gives us two equations of condition—

$$\nu(a - c\nu)\Delta\pi - \Delta\xi_0 + \xi - \xi_0 = 0$$

$$\nu(b - c\nu)\Delta\pi - \Delta\eta_0 + \eta - \eta_0 = 0$$

11. *The ephemerides.*—With very great kindness Professor Millosevich computed for me a heliocentric ephemeris of *Eros* for a part of the period during which it was under observation. My sincere acknowledgments are due to him for the readiness with which he undertook this indispensable part of the experiment.

This ephemeris, he writes, is founded upon the following elements :—

Osculation 1900 Oct. 31·5 B = epoch.

M	304° 24′ 49″·75
μ	2015″·23858
ϕ	12° 52′ 49″·93
ω	177° 39′ 8″·65
☊	303° 30′ 42″·38
i	10° 49′ 38″·03

log a 0·1637867.

These elements represent, account being taken of perturbations by *Venus*, *Earth*, *Mars*, *Jupiter*, and *Saturn*, within one second of arc, and generally within much less than that, all the normal places available between 1898 August and 1901 April.

Heliocentric ephemeris of Eros referred to the ecliptic of 1900·0.
Professor E. Millosevich.

12^h Berlin M.T. 1900.	Hel. Longitude. ° ′ ″	Hel. Latitude. ° ′ ″	Log. Rad. vect.
Oct. 19	33 44 48·31	+10 49 37·72	0·1373172
20	34 22 33·08	49 33·71	62860
21	35 0 28·64	49 25·06	52529
22	35 38 35·02	49 11·70	42182
23	36 16 52·26	48 53·58	31819
24	36 55 20·39	48 30·62	21442
25	37 33 59·48	48 2·76	11051
26	38 12 49·58	47 29·93	0·1300647
27	38 51 50·73	46 52·07	0·1290232
28	39 31 2·94	46 9·10	79808
29	40 10 26·27	45 20·98	69376
30	40 50 0·73	44 27·62	58938
31	41 29 46·52	43 28·97	48495
Nov. 1	42 9 43·47	42 24·95	38049
2	42 49 51·68	41 15·50	27600
3	43 30 11·20	40 0·55	17150
4	44 10 42·04	38 40·05	0·1206701
5	44 51 24·20	37 13·95	0·1196254
6	45 32 17·74	35 42·15	85811
7	46 13 22·71	34 4·56	75374
8	46 54 39·11	32 21·14	64945
9	47 36 6·93	30 31·86	54525
10	48 17 46·20	28 36·64	44117
11	48 59 36·96	26 35·41	33721
12	49 41 39·19	24 28·11	23339
13	50 23 52·89	22 14·68	12974
14	51 6 18·08	19 55·07	0·1102627
15	51 48 54·78	17 29·22	0·1092301
16	52 31 42·99	14 57·07	81998
17	53 14 42·70	12 18·56	71719
18	53 57 53·91	9 33·62	61466
19	54 41 16·64	6 42·22	51242
20	55 24 50·90	+10 3 44·28	0·1041048

The heliocentric ephemeris of the Earth is taken from the *Berliner Jahrbuch*, 1900.

Both these ephemerides are for the mean equinox 1900·0 of Leverrier's tables.

From them the ephemerides of the planet and the Earth expressed in rectangular coordinates referred to the first set of axes, through the centre of the Sun, were constructed with seven-figure logarithm tables.

12. *Formation of the equations of condition.*—The mean epochs of the exposures $T + \Delta T$ reduced to Berlin M.T. are given in Table I.

The values of ΔT, the light equation, were calculated from the values of $\log \Delta$ given by Millosevich in *A.N.* No. 3660, 1, using Gill's value of the Sun's light equation $498^s \cdot 46$. Thus T was found.

With these values of $T - 0^d \cdot 5$ (since the ephemeris of the planet is for midnight) and of $T + \Delta T$, the values of U, V, W, f, g, h were interpolated. The third differences are small and somewhat irregular, owing largely, no doubt, to the use of only seven-figure tables.

The values of the nine direction cosines required (see paragraph 10) are :—

l_1 $-\cdot 4891288$	m_1 $-\cdot 7089005$	n_1 $+\cdot 5081469$
l_2 $+\cdot 8001604$	m_2 $-\cdot 1328410$	n_2 $+\cdot 5848907$
l_3 $-\cdot 3471266$	m_3 $+\cdot 6926858$	n_3 $+\cdot 6322104$

The values of a, b, c were formed with the data for the Cambridge Observatory

$$\phi' = 52^\circ \ 1' \ 29'' \cdot 2$$

$$\log \rho'/\rho = \bar{1} \cdot 999078$$

It should be remarked that from this point no more logarithms were used. The whole of the rest of the calculations, as well as all the foregoing interpolations, were done on an arithmometer (the Thomas de Colmar), which calculates quantities of the form

$$l_1(U-f) + l_2(V-g) + l_3(W-h)$$

with great ease. It seems to me that this possibility of using an arithmometer almost throughout is a strong point of the method.

Table II. contains the quantities required in the formation of the equations of condition.

D

TABLE

Quantities required in the formation

Series A.

No.		T−0d·5	U	V	W	f	g	h
1	Nov.	9·050204	+ ·8641836	+ ·9476201	+ ·2378653	+ ·6731748	+ ·7258077	+ 5 × 10^{-7}
2		·103210	34709	80585	7979	24912	64246	5
3		·147103	28805	84212	7421	19248	69348	5
4		·186380	23520	87456	6920	14175	73911	6
5		·227238	·8618020	·9490828	·2376399	·6708895	·7278654	6
6		·774938	·8544107	·9535828	·2369364	·6637790	·7341874	9
7		·815713	38591	39163	8837	32472	46554	9
8		·854698	33315	42349	8332	27384	51025	9
9		9·961960	18790	51106	6940	13370	63309	9
10		10·013438	11814	55303	6271	06636	69196	10
11		·063477	·8505030	59380	5620	·6600085	74912	10
12		·103702	·8499575	62655	5096	·6594815	79503	10
13		10·147999	+ ·8493565	+ ·9566259	+ ·2364518	+ ·6589008	+ ·7384555	+ 10

Series B.

No.	T−0d·5	U	V	W	f	g	h
1	9·054790	+ ·8641220	+ ·9476581	+ ·2378595	+ ·6731157	+ ·7258612	+ 5 × 10^{-7}
2	·107279	34162	80921	7927	24387	64719	5
3	·151200	28254	84551	7368	18719	69825	5
4	·191109	22883	87846	6860	13564	74460	6
5	·231422	·8617456	·9491173	·2376346	·6708354	·7279139	6
6	·779209	·8543529	·9536177	·2369309	·6637233	·7342364	9
7	·820241	37978	39533	8778	31881	47073	9
8	·859344	32686	42729	8272	26777	51558	9
9	9·966371	18192	51466	6883	12793	63814	9
10	10·017362	11282	55623	6220	·6606122	69644	10
11	·067775	·8504447	59730	5564	·6599522	75403	10
12	·108001	·8498992	63005	5040	94252	79993	10
13	10·152671	+ 8492931	+ ·9566639	+ ·2364457	+ ·6588395	+ ·7385087	+ 10

II.

of the equations of condition.

L	M	N	$\nu(a-c\nu)$	$\nu(b-c\nu)$	ξ_0	η_0
+·00148839	−·00010645	+·37717714	+0·6742	+0·0153	+·00391740	−·00028288
138320	10879	701534	1·1249	0·2662	362088	29990
129628	11084	688159	1·4031	0·5539	339970	31770
121853	11306	676194	1·5613	0·8552	316769	33652
113768	11534	663763	+1·6248	1·1916	295138	35702
+·00006144	16176	498432	−1·5695	0·8430	023073	46730
−·00001820	16636	486228	1·3991	0·5318	+·00001107	46645
009430	17094	474564	1·1500	+0·2762	−·00020263	46792
030320	18428	442538	−0·1661	−0·1005	080269	48789
040333	19109	427188	+0·3633	−0·0748	109312	50737
050048	19789	412302	0·8427	+0·0802	137365	53236
057855	20359	400352	1·1700	0·2879	159677	55662
−·00066442	−·00021000	+·37387197	+1·4439	+0·5851	−·00183866	−·00058662

+·00147923	−·00010663	+·37716314	+0·7200	+0·0333	+·00389131	−·00028414
137511	10901	700292	1·1548	0·2901	359827	30151
128815	11111	686905	1·4236	0·5836	335736	31969
120917	11326	674752	1·5742	0·8932	314242	33869
112944	11552	662492	+1·6247	1·2498	292960	35998
+·00005310	16220	497153	−1·5564	0·8087	+·00020794	46703
−·00002700	16690	484871	1·3743	0·4996	−·00001346	46654
010338	17150	473178	1·1156	+0·2493	022834	46828
031180	18482	441223	−0·1209	−0·1037	082762	48920
041092	19164	426025	+0·4026	−0·0671	111511	50919
050884	19848	411021	0·8804	+0·0989	139765	53475
058683	20419	399077	1·2008	0·3139	162027	55936
−·00067343	−·00021072	+·37385815	+1·4664	+0·6201	−·00186379	−·00059007

Equations of condition.

	Series A.		n_1	n_2	v
ξ 1	$+0.67\Delta\pi$	$-1\Delta\xi_0$	-28×10^{-7}	$-\ 2\times10^{-7}=0$	$+\ 4\times10^{-7}$
2	$+1.12$	-1	-30	$-\ 6$	$-\ 1$
3	$+1.40$	-1	-39	-16	$-\ 9$
4	$+1.56$	-1	-22	0	$+\ 5$
5	$+1.62$	-1	$-\ 9$	$+12$	$+17$
6	-1.57	-1	$-\ 3$	$+\ 3$	$+\ 9$
7	-1.40	-1	-15	-10	$-\ 4$
8	-1.15	-1	-10	$-\ 6$	0
9	-0.17	-1	-11	-10	$-\ 4$
10	$+0.36$	-1	-14	-14	$-\ 8$
11	$+0.84$	-1	$-\ 7$	$-\ 9$	$-\ 3$
12	$+1.17$	-1	$-\ 2$	$-\ 5$	0
13	$+1.44$	-1	$-\ 7$	-11	$-\ 6$
η 1	$+0.02$	$-1\Delta\eta_0$	$+71$	$+67$	$-\ 1$
2	$+0.27$	-1	$+71$	$+67$	$-\ 1$
3	$+0.55$	-1	$+79$	$+76$	$+\ 8$
4	$+0.86$	-1	$+83$	$+80$	$+12$
5	$+1.19$	-1	$+59$	$+56$	-12
6	$+0.84$	-1	$+68$	$+67$	$-\ 1$
7	$+0.53$	-1	$+73$	$+72$	$+\ 4$
8	$+0.28$	-1	$+61$	$+60$	$-\ 8$
9	-0.10	-1	$+67$	$+67$	$-\ 1$
10	-0.07	-1	$+66$	$+66$	$-\ 2$
11	$+0.08$	-1	$+69$	$+69$	$+\ 1$
12	$+0.29$	-1	$+69$	$+69$	$+\ 1$
13	$+0.59$	-1	$+70$	$+71$	$+\ 3$

	Series B.		n_1	n_2	v
ξ 1	$+0.72\Delta\pi$	$-1\Delta\xi_0$	-25×10^{-7}	$-\ 4\times10^{-7}=0$	$+\ 2\times10^{-7}$
2	$+1.15$	-1	-33	-13	$-\ 9$
3	$+1.42$	-1	-26	$-\ 7$	$-\ 3$
4	$+1.57$	-1	-25	$-\ 7$	$-\ 4$
5	$+1.62$	-1	$-\ 4$	$+13$	$+16$
6	-1.56	-1	-17	-12	$+\ 2$
7	-1.37	-1	-23	-19	$-\ 6$
8	-1.12	-1	$-\ 6$	$-\ 3$	$+\ 9$
9	-0.12	-1	$-\ 7$	$-\ 6$	$+\ 3$
10	$+0.40$	-1	$-\ 4$	$-\ 4$	$+\ 3$

	Series B.		n_1	n_2	v
11	$+0{\cdot}88\Delta\pi - 1\Delta\xi_0$		-10×10^{-7}	$-12\times10^{-7}=0$	-7×10^{-7}
12	$+1{\cdot}20$	-1	-3	-5	-1
13	$+1{\cdot}47$	-1	-5	-8	-5
η 1	$+0{\cdot}03$	$-1\Delta\eta_0$	$+68$	$+60$	-5
2	$+0{\cdot}29$	-1	$+72$	$+65$	-1
3	$+0{\cdot}58$	-1	$+80$	$+71$	$+4$
4	$+0{\cdot}89$	-1	$+74$	$+66$	-2
5	$+1{\cdot}25$	-1	$+88$	$+82$	$+13$
6	$+0{\cdot}81$	-1	$+67$	$+65$	-3
7	$+0{\cdot}50$	-1	$+64$	$+63$	-4
8	$+0{\cdot}25$	-1	$+61$	$+60$	-6
9	$-0{\cdot}10$	-1	$+62$	$+62$	-3
10	$-0{\cdot}07$	-1	$+70$	$+70$	$+5$
11	$+0{\cdot}10$	-1	$+66$	$+67$	$+2$
12	$+0{\cdot}31$	-1	$+61$	$+62$	-4
13	$+0{\cdot}62$	-1	$+69$	$+70$	$+3$

The numerical terms which arise from the combination of the quantities of Table II. are found in the column n_1. They are expressed in terms of the length of the radius of projection on to the plate; that is to say, they are independent of the focal length of the particular telescope employed, so that the equations could be combined immediately with the work of other telescopes reduced in the same way.

13. *Empirical correction of the ephemeris.*—A glance at the terms in column n_1 of the ξ equations shows that the equations cannot be satisfied as they stand. We must introduce a correction to the ephemeris varying with the time.

An approximate value of this correction was found as follows:

Taking groups of equations of condition which have similar parallax factors on the two nights, and writing them in the form

$$a\Delta\pi + \Delta\xi_0 + \Delta'\xi_0 t + n = 0$$

we have from among the ξ equations of Series A:

1	$+0{\cdot}67\ \Delta\pi + \Delta\xi_0$	$+0{\cdot}05\ \Delta'\xi_0$	$-28=0$
2	$+1{\cdot}12$	$+0{\cdot}10$	-30
3	$+1{\cdot}40$	$+0{\cdot}15$	-39
11	$+0{\cdot}84$	$+1{\cdot}06$	-7
12	$+1{\cdot}17$	$+1{\cdot}10$	-2
13	$+1{\cdot}44$	$+1{\cdot}15$	-7

and subtracting the mean of the first three from the mean of the second

$$0{\cdot}09\ \Delta\pi + 1{\cdot}00\ \Delta'\xi_0 + 27 = 0.$$

The effect of $\Delta\pi$ is inappreciable. Proceeding in the same way with the other groups we find that the corrections to be added to the numerical terms to allow for an error in the ephemeris varying with the time are

$$\Delta'\xi_0 = \text{Series A } -27t \qquad \text{Series B } -22t$$
$$\Delta'\eta_0 = \qquad +\ 4t \qquad\qquad +\ 8t$$

Reckoning t from Berlin midnight on November 10, the corrected values of the numerical terms are found in the column n_2 of the equations of condition.

This does not, however, necessarily represent a real correction to the assumed motion of the planet. The greater part may be due to the accumulation of error in adding two separate ephemerides each computed with seven-figure logarithm tables. The necessity for it should disappear with more accurate ephemerides. It has been used here as a temporary expedient, and it is practicable only when we have observations on two consecutive nights at the same hour-angles, which was not a common case last winter.

14. *Solution of the equations of condition.*—In this preliminary solution equal weight has been given to each equation.

The normal equations from the twenty-six equations of condition of Series A are :

$$+22{\cdot}83\ \Delta\pi - 5{\cdot}89\ \Delta\xi_0 - 5{\cdot}33\ \Delta\eta_0 + 336{\cdot}59 = 0$$
$$-\ 5{\cdot}89\ \Delta\pi + 13{\cdot}00\ \Delta\xi_0 \quad 0{\cdot}00\ \Delta\eta_0 + \ 74{\cdot}00 = 0$$
$$-\ 5{\cdot}33\ \Delta\pi \quad 0{\cdot}00\ \Delta\xi_0 + 13{\cdot}00\ \Delta\eta_0 - 887{\cdot}00 = 0$$

And their solution gives

P.E. of one equation	$\pm 4{\cdot}63$	or	$\pm 0''{\cdot}096$	Wt.
$\Delta\pi = -\ 0{\cdot}36$	$\pm 1{\cdot}09$	,,	$-0''{\cdot}007 \pm 0''{\cdot}023$	17·97
$\Delta\xi_0 = -\ 5{\cdot}85$	$\pm 1{\cdot}38$	,,	$-\ 0{\cdot}12 \pm 0{\cdot}028$	11·32
$\Delta\eta_0 = +68{\cdot}08$	$\pm 1{\cdot}36$	,,	$+\ 1{\cdot}40 \pm 0{\cdot}028$	11·59

Similarly for Series B :

$$+23{\cdot}31\ \Delta\pi - 6{\cdot}26\ \Delta\xi_0 - 5{\cdot}46\ \Delta\eta_0 + 381{\cdot}65 = 0$$
$$-\ 6{\cdot}26\ \Delta\pi + 13{\cdot}00\ \Delta\xi_0 \quad 0{\cdot}00\ \Delta\eta_0 + \ 87{\cdot}00 = 0$$
$$-\ 5{\cdot}46\ \Delta\pi \quad 0{\cdot}00\ \Delta\xi_0 + 13{\cdot}00\ \Delta\eta_0 - 863{\cdot}00 = 0$$

P.E. of one equation	$\pm 4{\cdot}26$	or	$\pm 0''{\cdot}088$	Wt.
$\Delta\pi = -\ 3{\cdot}40$	$\pm 1{\cdot}01$	,,	$-0''{\cdot}070 \pm\ 0{\cdot}021$	18·00
$\Delta\xi_0 = -\ 8{\cdot}33$	$\pm 1{\cdot}28$	,,	$-\ 0{\cdot}17 \pm\ 0{\cdot}026$	11·13
$\Delta\eta_0 = +64{\cdot}85$	$\pm 1{\cdot}25$	,,	$+\ 1{\cdot}34 \pm\ 0{\cdot}026$	11·53

The residuals which arise from substituting these values of the unknowns in the equations of condition are given in the last column of the table. It will be noticed that the residuals in the four equations No. 5, which represent the last set of exposures on November 9, are unduly large. I have remeasured the images of the planet and examined the reductions, and can find no error. The zenith distance was about 50° and the images are bad; indeed throughout the whole of these two nights the seeing was not at all good; they were the first frosty nights of the winter.

It does not seem necessary to give the value of π_0 which has been assumed in these reductions, as it is inadvisable to multiply results which are in no sense definitive, even for the small number of observations here discussed.

15. The difference $0''\cdot06$ between the results of Series A (stars fixed) and Series B (planet fixed) looks somewhat large, though it is well within the possible difference of two results each with a P.E. $\pm 0''\cdot02$. For the present I should prefer to look upon it as bad luck, and should deprecate the drawing of any conclusion confirmatory of the fears that have been expressed that systematic error may arise from the motion of the planet. If such error were independent of the magnitude, and a function only of the motion of the planet and the time of exposure—that is to say, of the length of the trail—each series would give a value of the parallax free from systematic error, since the time of exposure and the motion of the planet were sensibly constant for the two nights; and both would give similarly erroneous values of the tabular corrections. If the error were a function of the magnitude, which varies effectively with the zenith distance, it should nevertheless be eliminated by the method here employed of reducing by means of comparison stars very nearly equal to the planet in magnitude. Moreover some evidence has been given that, even for stars on the limit of photographic action for the particular exposure, no certain trace of the effects of trail is discoverable. I prefer then to draw no further conclusion from the discrepancy than that it will be wise to adhere to my resolution to keep the two series separate right through the work.

16. *Tabular errors of the planet.*—It is a not inconsiderable advantage of this method that we can easily derive from the values found for $\Delta\xi_0$, $\Delta\eta_0$, the corrections to the tabular heliocentric longitude and latitude of the planet, if we assume that the ephemeris of the Earth is accurate.

From the relations between U, V, W and λ, β, R, the heliocentric longitude, latitude, and radius vector, we have

$$\begin{aligned}
\Delta U &= \cos\beta\cos\lambda \,.\, \Delta R - R\cos\beta\sin\lambda \,.\, \Delta\lambda - R\cos\lambda\sin\beta \,.\, \Delta\beta \\
\Delta V &= \cos\beta\sin\lambda \,.\, \Delta R + R\cos\beta\cos\lambda \,.\, \Delta\lambda - R\sin\lambda\sin\beta \,.\, \Delta\beta \\
\Delta W &= \sin\beta \,.\, \Delta R + R\cos\beta \,.\, \Delta\beta
\end{aligned}$$

And we had

$$\nu\,\Delta\xi_0 = l_1\,\Delta U + l_2\,\Delta V + l_3\,\Delta W$$
$$\nu\,\Delta\eta_0 = m_1\,\Delta U + m_2\,\Delta V + m_3\,\Delta W$$

Putting in the values of β, λ, R, l_1 . . . &c. we obtain two equations for $\Delta\lambda$ and $\Delta\beta$ in terms of ΔR:

$$1{\cdot}148\,\Delta\lambda - 0{\cdot}509\,\Delta\beta = -\;1{\cdot}89 - 0{\cdot}204\,\Delta R$$
$$+0{\cdot}564\,\Delta\lambda + 1{\cdot}021\,\Delta\beta = +17{\cdot}73 + 0{\cdot}435\,\Delta R$$

whence

$$\Delta\lambda = +\;4{\cdot}9 + 0{\cdot}009\,\Delta R$$
$$\Delta\beta = +14{\cdot}7 + 0{\cdot}421\,\Delta R.$$

ΔR must be derived from an extended comparison with observation of the theory of the planet's motion. Putting it zero for the present, we have $\Delta\lambda = +0''{\cdot}10$ $\Delta\beta = +0''{\cdot}30$.

There is, therefore, no need to introduce a geocentric ephemeris of the planet at any point of the work. I have used the values of log Δ from Millosevich's ephemeris to get the light equation, because they were ready to hand. But the geocentric distances of the planet are easily found from

$$\Delta^2 = (U-f)^2 + (V-g)^2 + (W-h)^2.$$

17. *Conclusions.*—To sum up, I should suggest that the following conclusions may be drawn from the results of this experimental reduction of a small series of *Eros* plates.

(1) The smallness of the probable error of an equation of condition, even with a comparatively rough ephemeris, is a good omen for the ultimate success of the enterprise.

(2) It is an absolute necessity to have ephemerides computed with eight-figure logarithms. This is indeed a truism, for since Sir David Gill found that it was necessary for his heliometer determination of the solar parallax, we should be wasting time now if our results were not good enough to demand it.

(3) There is only one complete set of eight-figure tables in existence—viz. those published by the French *Service Géographique de l'Armée*, which are based on the centesimal division of the quadrant. *We must therefore work in the centesimal system.* Professor Bauschinger kindly informs me that the ephemeris of Victoria for Sir David Gill was computed in the K. Rechen Institut, Berlin, in grades, and afterwards converted into degrees, "was einige lästige Mehrarbeit verursacht." There does not seem to be any good reason why we should give ourselves this extra labour.

(4) I hope that the present reduction has sufficiently shown the advantages which arise from working with the separate heliocentric ephemerides of the Earth and *Eros*, instead of with the usual geocentric ephemeris of the planet. Briefly, they are

(*a*) the ephemeris of the Earth can be computed once for all; the ephemeris of the planet can with comparatively small labour be improved from time to time as we proceed, and the corrections can be carried into our equations of condition with the minimum of trouble. (*b*) The interpolations from the heliocentric ephemerides are not so troublesome as from a geocentric; the quantities vary more regularly. (*c*) The greater part of the calculation is done on a machine instead of with logarithms, which saves a deal of writing. (*d*) The form of the work is comparatively simple, and you see clearly what you are doing all through.

(5) Since the sub-committee of the *Comité International Permanent* is publishing the revised places of the standard stars referred to Newcomb's Fundamental Catalogue, it will be necessary to refer the elements of the orbit of *Eros* to Newcomb's system—at present they are based on Leverrier's—and to compute the ephemeris of the Earth from Newcomb's tables of the Sun, with Gill's value for the mass of the Moon.

(6) Finally, the great advantage is that photographic results reduced on these lines by different observatories would be most readily available for a general combination. And the micrometric results could be easily included, for they are essentially measures in rectangular coordinates, and they might be reduced in exactly the same way.

Cambridge Observatory:
1901 *November* 6.

The Determination of Selenographic Positions and the Measurement of Lunar Photographs.

[Second Paper.]

Determination of a first group of Standard Points by Measures made at the Telescope and on Photographs. By S. A. Saunder, M.A.

§ 1. *Recapitulation of First Paper.*

In a previous paper (*Monthly Notices*, vol. lx. p. 174) I called attention to the unsatisfactory state of our knowledge of the exact positions of the lunar formations, and to the increase in accuracy which might be obtained by measuring from the well-determined point *Mösting A*; formulæ were developed for reducing the measures, and a few results were given.

It was also shown that by the measurement of such photographs as those now being taken at the Paris Observatory a great increase might be made in the number of points whose positions could be accurately determined without necessitating